

The Feasibility of Interference Alignment for Reverse TDD Systems in MIMO Cellular Networks

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Abstract—The feasibility conditions of interference alignment (IA) are analyzed for reverse TDD systems, i.e., one cell operates as downlink (DL) but the other cell operates as uplink (UL). Under general multiple-input and multiple-output (MIMO) antenna configurations, a necessary condition and a sufficient condition for one-shot linear IA are established, i.e., linear IA without symbol or time extension. In several example networks, optimal sum degrees of freedom (DoF) is characterized by the derived necessary condition and sufficient condition. For symmetric DoF within each cell, a sufficient condition is established in a more compact expression, which yields the necessary and sufficient condition for a class of symmetric DoF. An iterative construction of transmit and received beamforming vectors is further proposed, which provides a specific beamforming design satisfying one-shot IA. Simulation results demonstrate that the proposed IA not only achieve larger DoF but also significantly improve the sum rate in the practical signal-to-noise ratio (SNR) regime.

Index Terms—Degrees of freedom, dynamic TDD, feasibility conditions, heterogeneous networks, interference alignment, multiple-input and multiple-output (MIMO), reverse TDD.

I. INTRODUCTION

As various types of wireless devices and networks emerged, wireless communication demands are explosively increasing and becoming diverse for providing an integrated service of voice, data, video, and so on [1], [2]. In order to support such demands, standard groups for next-generation mobile communication have actively studied heterogeneous networks (HetNets) consisting of different types communication systems and devices with different capabilities [3]–[9]. In particular, dynamic time division duplex (D-TDD) is considered as a promising technology for heterogeneous cellular networks, which dynamically adjusts the portion of uplink (UL) and downlink (DL) frames based on the current communication traffic and environment in a distributed manner

by each heterogeneous cell [10]–[15]. Based on the same principle, enhanced interference mitigation and traffic adaptation (eIMTA) has been studied in the 3GPP standard group [16].

Unlike the conventional cellular networks in which UL and DL phases are synchronized over the entire cells, dynamic adaptation of UL and DL in each cell applied in D-TDD and eIMTA essentially requires a *new interference mitigation technique between asynchronous UL and DL cells*. Specifically, for a given time slot, UL cells and DL cells may coexist in a network because of asynchronous coordination between cells, which is also referred to as reverse TDD (R-TDD) systems. For such R-TDD systems, due to limited capabilities of terminals or users, interference mitigation from the users in UL cells to the users in DL cells is quite challenging, but crucially important for boosting spectral efficiency. In the context of D-TDD or R-TDD systems, various interference mitigation techniques have been actively studied in the literature, see [5]–[12] and the references therein. For instances, R-TDD techniques has been studied in HetNets to achieve improved area spectral efficiency [5] or improved throughputs [6]–[8]. To suppress strong interferences from adjacent users, scheduling policies for D-TDD systems have been studied in [10]–[12]. The basic principle behind these interference management techniques is to *avoid strong interferences by orthogonalization and treat weak interferences as noise*.

Cadambe and Jafar recently made a remarkable progress showing that the sum degrees of freedom (DoF) of the K -user interference channel (IC) is given by $K/2$ [17]. A new interference mitigation paradigm called *interference alignment* (IA) has been proposed to achieve $K/2$ DoF, which align interfering signals from multiple transmitters into the same signal space. The concept of this signal space alignment has been successfully adapted to various network environments, e.g., see [18]–[24] and the the references therein. Different strategies of IA were also developed under the name of ergodic IA [25]–[28] and real IA [29], [30]. In [19], [20], IA for cellular networks has been studied for both UL and DL scenarios showing that multiple users in each cell are beneficial for improving DoF. More recently, IA techniques using multiple antennas have been actively studied in order to boost DoF of multiantenna multiuser networks [31]–[51]. In particular, IA exploiting multiple antennas has been studied in both cellular UL [38]–[43] and DL [44]–[51].

In spite of the rapid advances on IA studies for the conventional cellular networks in which the entire cells operate either UL or DL [38]–[51], relatively little progress has been made so far on IA for R-TDD systems having the coexistence of UL

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and DL cells. One notable work done by Jeon and Suh is to study IA for two-cell R-TDD systems showing that, depending on the antenna configuration, operating one cell as UL and the other cell as DL can enlarge the sum DoF of multiantenna cellular networks than the conventional UL or DL operation [52]. To achieve such DoF gain, asymptotic signal space IA is needed for aligning multiple interfering signals transmitted from the UL users at each of the DL users, which has to apply transmit beamforming over multiple channel instances [18]–[21], [23], [24].

In many practical cellular networks, however, asymptotic signal space IA is hard to implement due to system complexity and feedback overhead. Furthermore, it requires large enough channel diversity to improve DoF, which may cause severe delay when the channel coherence time is large. In order to overcome such limitations, *single-shot IA using multiple antennas*, i.e., multiantenna beamforming using single time instance, has been considered for the K -user IC [33]–[37] and also for cellular networks [42]–[45]. The feasibility conditions of single-shot IA has been first established in [33], [34] for the K -user multiple-input and multiple-output (MIMO) IC having M and N antennas at each transmitter and the receiver respectively. Followed by [33], [34], a more general antenna configuration has been considered in [35], [36]. In particular, [35] established a tighter necessary condition and also the necessary and sufficient condition for a class of network configurations and [36] provided a sufficient condition for general network configurations. More recently, study on IA feasibility has been extended to MIMO cellular networks for both UL [42], [43] and DL [44], [45]. IA feasibility conditions have been established in [42], [43] for various UL settings and in [44], [45] for MIMO cellular networks assuming DL. In spite of recent demand on single-shot IA for R-TDD systems along with its significance to heterogeneous cellular networks, the previous work on IA feasibility for MIMO cellular networks inherently assumed cellular UL or DL, i.e., the entire cells operate either UL or DL.

In this paper, we study *IA feasibility for R-TDD cellular networks having multiple antennas at both base stations (BSs) and users*. We focus on two-cell environment in which one cell operates as UL but the other cell operates as DL, and restrict to linear IA coding scheme without time or symbol extension as the same reasons assumed in [33]–[37], [42]–[44]. The main contributions of this paper are as follows.

- We derive a necessary condition and a sufficient condition on one-shot linear IA for a general MIMO antenna configurations. In several example networks, optimal sum DoF is characterized by the established necessary condition and sufficient condition. We further demonstrate that the proposed IA improves DoF than that without IA between UL and DL cells.
- For a symmetric DoF in each cell, we establish a sufficient condition on one-shot linear IA with a more compact expression. From the sufficient condition derived with a more compact expression, we establish the necessary and sufficient condition for a class of symmetric DoF.
- We provide an iterative construction method of precod-

ing and postcoding matrices, which provides a specific beamforming design at finite signal-to-noise ratio (SNR). Simulation results demonstrate that the proposed IA not only achieve better DoF but also significantly improve the sum rate in the practical SNR regime.

The rest of this paper is organized as follows. In Section II, we introduce the considered MIMO R-TDD cellular network and formally define the IA feasibility problem. In Section III, we first state the main results, i.e., a necessary condition and a sufficient condition established in the paper and provide several example networks for better understanding of the main results. The detailed proofs of the main results are provided in Section IV. In Section V, we propose an iterative construction method for precoding and postcoding matrices and demonstrate by simulation that the proposed construction can improve the sum rate of R-TDD systems in the practical SNR regime. We finally conclude in Section VI.

II. PROBLEM FORMULATION

In this section, we explain the notation used in the paper and introduce the considered MIMO R-TDD cellular network. We then formally define the feasibility problem for one-shot linear IA.

A. Notation

Let us introduce the notation used in the paper. For a matrix \mathbf{A} , denote the i th row vector and the (i, j) th element of \mathbf{A} by $\mathbf{A}[i]$ and $\mathbf{A}[i, j]$ respectively. Also, \mathbf{A}^\dagger , \mathbf{A}^T , $\|\mathbf{A}\|$, $\text{rank}(\mathbf{A})$, and $\det(\mathbf{A})$ denote the Hermitian transpose, transpose, Frobenius norm, rank, and determinant of \mathbf{A} respectively. The operator $\text{vec}(\mathbf{A})$ converts \mathbf{A} into the column vector constructed by stacking the column vectors of \mathbf{A} , $\text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_n)$ denotes the block diagonal matrix whose diagonal blocks are given by $\mathbf{A}_1, \dots, \mathbf{A}_n$, and $\text{diag}[n](\mathbf{A})$ denotes the block diagonal matrix whose diagonal blocks are given by \mathbf{A} for n times. The identity matrix of size n is denoted by \mathbf{I}_n and the all-zero matrix of size $m \times n$ is denoted by $\mathbf{0}_{m \times n}$. For a set \mathcal{A} , denote its cardinality by $|\mathcal{A}|$. The set of complex numbers and the set of natural numbers are denoted by \mathbb{C} and \mathbb{N} respectively. Let $\text{mod}(n, m)$ denote the modulo operation, i.e., the remainder of n divided by m . The multivariate complex Gaussian distribution with a mean vector \mathbf{m} and a covariance matrix \mathbf{C} is denoted by $\mathcal{CN}(\mathbf{m}, \mathbf{C})$.

B. MIMO Reverse Time Division Duplex Cellular Networks

Consider a MIMO R-TDD cellular network depicted in Fig. 1 in which the first cell (cell α) operates as DL, while the second cell (cell β) operates as UL. In particular, BS α equipped with M_α antennas wishes to transmit independent messages to its K serving users, labeled user $(\alpha, 1)$ to user (α, K) . On the other hand, BS β equipped with M_β antennas wishes to receive independent messages from its L serving users, labeled user $(\beta, 1)$ to user (β, L) . User (α, k) is equipped with $N_{\alpha k}$ antennas and user (β, l) is equipped with $N_{\beta l}$ antennas, where $k \in [1 : K]$ and $l \in [1 : L]$.

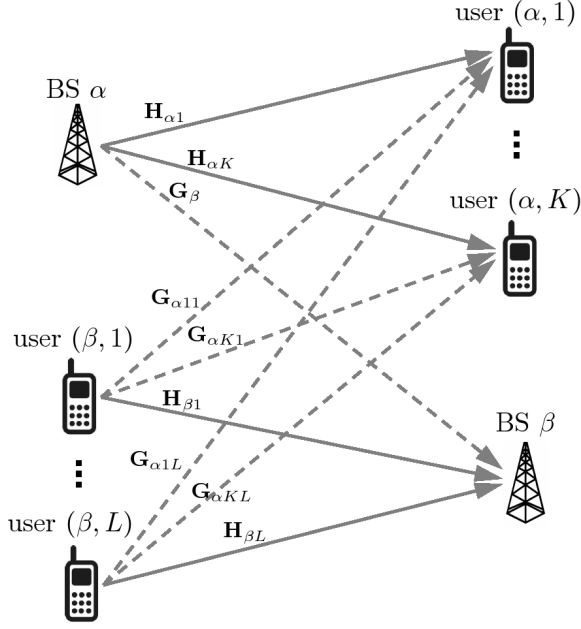


Fig. 1. A network model of MIMO R-TDD cellular networks

The received signal vector of user (α, k) is given by

$$\mathbf{y}_{\alpha k} = \mathbf{H}_{\alpha k} \mathbf{x}_{\alpha} + \sum_{l=1}^L \mathbf{G}_{\alpha k l} \mathbf{x}_{\beta l} + \mathbf{z}_{\alpha k} \quad (1)$$

for $k \in [1 : K]$ and the received signal vector of BS β is given by

$$\mathbf{y}_{\beta} = \sum_{l=1}^L \mathbf{H}_{\beta l} \mathbf{x}_{\beta l} + \mathbf{G}_{\beta} \mathbf{x}_{\alpha} + \mathbf{z}_{\beta} \quad (2)$$

where $\mathbf{H}_{\alpha k} \in \mathbb{C}^{N_{\alpha k} \times M_{\alpha}}$, $\mathbf{G}_{\alpha k l} \in \mathbb{C}^{N_{\alpha k} \times N_{\beta l}}$, $\mathbf{H}_{\beta l} \in \mathbb{C}^{M_{\beta} \times N_{\beta l}}$, and $\mathbf{G}_{\beta} \in \mathbb{C}^{M_{\beta} \times M_{\alpha}}$ are the channel matrices from BS α to user (α, k) , from user (β, l) to user (α, k) , from user (β, l) to BS β , and from BS α to BS β , respectively. Also, $\mathbf{x}_{\alpha} \in \mathbb{C}^{M_{\alpha} \times 1}$ is the transmit signal vector of BS α and $\mathbf{x}_{\beta l} \in \mathbb{C}^{N_{\beta l} \times 1}$ is the transmit signal vector of user (β, l) . The additive noise vectors $\mathbf{z}_{\alpha k} \in \mathbb{C}^{N_{\alpha k} \times 1}$ and $\mathbf{z}_{\beta} \in \mathbb{C}^{M_{\beta} \times 1}$ are assumed to follow $\mathcal{CN}(\mathbf{0}_{N_{\alpha k} \times 1}, \mathbf{I}_{N_{\alpha k}})$ and $\mathcal{CN}(\mathbf{0}_{M_{\beta} \times 1}, \mathbf{I}_{M_{\beta}})$, respectively. BS α and each user in cell β should satisfy the average power constraint, i.e., $\mathbb{E}(\|\mathbf{x}_{\alpha}\|^2) \leq P_{\alpha}$ and $\mathbb{E}(\|\mathbf{x}_{\beta l}\|^2) \leq P_{\beta l}$ for all $l \in [1 : L]$. We assume that all channel coefficients are independent and identically distributed (i.i.d.) from a continuous distribution. Global channel state information is assumed to be available at each user and BS.

For notational simplicity, denote the considered MIMO R-TDD cellular network as the $(M_{\alpha}, (N_{\alpha 1}, \dots, N_{\alpha K})) \times (M_{\beta}, (N_{\beta 1}, \dots, N_{\beta L}))$ MIMO R-TDD cellular network.

C. Feasibility for Linear Interference Alignment

Suppose that BS α sends $d_{\alpha k} \in \mathbb{N}$ independent streams, denoted by $\mathbf{s}_{\alpha k} \in \mathbb{C}^{d_{\alpha k} \times 1}$, to user (α, k) using the precoding matrix $\mathbf{V}_{\alpha k} \in \mathbb{C}^{M_{\alpha} \times d_{\alpha k}}$ and user (β, l) sends $d_{\beta l} \in \mathbb{N}$ independent streams, denoted by $\mathbf{s}_{\beta l} \in \mathbb{C}^{d_{\beta l} \times 1}$, to BS β using

the precoding matrix $\mathbf{V}_{\beta l} \in \mathbb{C}^{N_{\beta l} \times d_{\beta l}}$. That is,

$$\mathbf{x}_{\alpha} = \sum_{k=1}^K \mathbf{V}_{\alpha k} \mathbf{s}_{\alpha k}, \quad (3)$$

$$\mathbf{x}_{\beta l} = \mathbf{V}_{\beta l} \mathbf{s}_{\beta l} \quad (4)$$

for $l \in [1 : L]$. Then user (α, k) estimates $\mathbf{s}_{\alpha k}$ using the postcoding matrix $\mathbf{U}_{\alpha k} \in \mathbb{C}^{N_{\alpha k} \times d_{\alpha k}}$ and BS β estimates $\mathbf{s}_{\beta l}$ using the postcoding matrix $\mathbf{U}_{\beta l} \in \mathbb{C}^{M_{\beta} \times d_{\beta l}}$. That is,

$$\begin{aligned} \hat{\mathbf{s}}_{\alpha k} &= \mathbf{U}_{\alpha k}^{\dagger} \mathbf{y}_{\alpha k} \\ &= \sum_{i=1}^K \mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha i} \mathbf{s}_{\alpha i} + \sum_{l=1}^L \mathbf{U}_{\alpha k}^{\dagger} \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l} \mathbf{s}_{\beta l} + \mathbf{U}_{\alpha k}^{\dagger} \mathbf{z}_{\alpha k} \end{aligned} \quad (5)$$

for all $k \in [1 : K]$ and

$$\begin{aligned} \hat{\mathbf{s}}_{\beta l} &= \mathbf{U}_{\beta l}^{\dagger} \mathbf{y}_{\beta} \\ &= \sum_{j=1}^L \mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\beta j} \mathbf{V}_{\beta j} \mathbf{s}_{\beta j} + \sum_{k=1}^K \mathbf{U}_{\beta l}^{\dagger} \mathbf{G}_{\beta} \mathbf{V}_{\alpha k} \mathbf{s}_{\alpha k} + \mathbf{U}_{\beta l}^{\dagger} \mathbf{z}_{\beta} \end{aligned} \quad (6)$$

for all $l \in [1 : L]$, where the second equality in (5) follows from (1) and (3) and the second equality in (6) follows from (2) and (4).

In similar manners in [33]–[37], [42]–[44], from (5) and (6), we define the feasible problem of one-shot linear IA for the MIMO R-TDD cellular network as follows.

Definition 1 (IA feasibility conditions): For the MIMO R-TDD cellular network, one-shot linear IA is said to be feasible if there exist $\{\mathbf{U}_{\alpha k}, \mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}, \mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying the following set of conditions:

$$\mathbf{U}_{\alpha k}^{\dagger} \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l} = \mathbf{0}, \quad \forall k, l, \quad (7a)$$

$$\mathbf{U}_{\beta l}^{\dagger} \mathbf{G}_{\beta} \mathbf{V}_{\alpha k} = \mathbf{0}, \quad \forall k, l, \quad (7b)$$

$$\mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha i} = \mathbf{0}, \quad \forall i \neq k, \quad (7c)$$

$$\mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\beta j} \mathbf{V}_{\beta j} = \mathbf{0}, \quad \forall j \neq l, \quad (7d)$$

$$\text{rank}(\mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha k}) = d_{\alpha k}, \quad \forall k, \quad (7e)$$

$$\text{rank}(\mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\beta l} \mathbf{V}_{\beta l}) = d_{\beta l}, \quad \forall l \quad (7f)$$

where $i, k \in [1 : K]$ and $j, l \in [1 : L]$.¹

Notice that (7a) to (7d) correspond to inter-cell IA from the users in cell β to the users in cell α , inter-cell IA from BS α to BS β , intra-cell IA within the users in cell α , inter-cell IA within the users in cell β , respectively. Assuming the IA feasibility conditions in (7a) to (7d), (7e) is required to guarantee that one DoF is able to be delivered by each stream in cell α and (7f) is required to guarantee that one DoF is able to be delivered by each stream in cell β .

Throughout the paper, we will simply state that $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$ is feasible if there is a feasible one-shot linear IA solution satisfying the set of conditions in

¹For notational simplicity, we drop the subscript in $\mathbf{0}_{n \times m}$ when the size of all-zero matrices are clear from the context.

Definition 1 with the given $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$. Obviously, user (α, k) is able to achieve $d_{\alpha k}$ DoF and user (β, l) is able to achieve $d_{\beta l}$ DoF in this case. Denote the feasible sum DoF as

$$d_{\text{sum}} = \sum_{k=1}^K d_{\alpha k} + \sum_{l=1}^L d_{\beta l}. \quad (8)$$

In the rest of the paper, we will analyze a necessary condition and a sufficient condition on the IA feasibility for MIMO R-TDD cellular networks described in Definition 1. For a feasible d_{sum} , we state that it is optimal in the following sense.

Definition 2 (Optimal d_{sum}): The sum DoF d_{sum} is said to be optimal if there exist a necessary condition and a sufficient condition on the IA feasibility in Definition 1 such that d_{sum} is the maximum sum DoF not just satisfying the necessary condition but also satisfying the sufficient condition at the same time.

Note that the optimal d_{sum} in Definition 2 is the maximum sum DoF achievable by all possible single-shot linear IA strategies.

Remark 1 (Frequency division duplex (FDD) systems): Although we state the IA feasibility based on TDD systems, the main results in this paper also hold for FDD systems.

III. MAIN RESULTS

In this section, we state our main results. We establish a necessary condition and a sufficient condition on the IA feasibility for MIMO R-TDD cellular networks. We then provide several example networks that their optimal d_{sum} are characterized by the derived necessary condition and sufficient condition. These example networks also demonstrate that the proposed one-shot linear IA using multiple antennas is beneficial for improving the sum DoF of MIMO R-TDD cellular networks.

Theorem 1 (Necessary condition): For the MIMO R-TDD cellular network, any feasible $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$ must satisfy the following set of conditions:

$$\sum_{k=1}^K d_{\alpha k} \leq M_{\alpha}, \quad (9a)$$

$$\sum_{l=1}^L d_{\beta l} \leq M_{\beta}, \quad (9b)$$

$$\sum_{k=1}^K d_{\alpha k} + \sum_{l=1}^L d_{\beta l} \leq \max(M_{\alpha}, M_{\beta}), \quad (9c)$$

$$\sum_{k \in \mathcal{I}_{\alpha}} d_{\alpha k} + \sum_{l \in \mathcal{I}_{\beta}} d_{\beta l} \leq \max\left(\sum_{k \in \mathcal{I}_{\alpha}} N_{\alpha k}, \sum_{l \in \mathcal{I}_{\beta}} N_{\beta l}\right), \quad \forall \mathcal{I}_{\alpha}, \mathcal{I}_{\beta}, \quad (9d)$$

$$\begin{aligned} \sum_{k \in \mathcal{I}_{\alpha}} \sum_{l \in \mathcal{I}_{\beta}} d_{\alpha k} d_{\beta l} &\leq \sum_{k \in \mathcal{I}_{\alpha}} d_{\alpha k} (N_{\alpha k} - d_{\alpha k}) \\ &\quad + \sum_{l \in \mathcal{I}_{\beta}} d_{\beta l} (N_{\beta l} - d_{\beta l}), \quad \forall \mathcal{I}_{\alpha}, \mathcal{I}_{\beta} \end{aligned} \quad (9e)$$

where $\mathcal{I}_{\alpha} \subseteq [1 : K]$ and $\mathcal{I}_{\beta} \subseteq [1 : L]$.

Proof: We refer to Section IV-A for the proof. ■

In order to state a sufficient condition, partition $\mathbf{G}_{\alpha kl}$ into four sub matrices as

$$\mathbf{G}_{\alpha kl} = \begin{bmatrix} \mathbf{G}_{\alpha kl}^{(1)} & \mathbf{G}_{\alpha kl}^{(2)} \\ \mathbf{G}_{\alpha kl}^{(3)} & \mathbf{G}_{\alpha kl}^{(4)} \end{bmatrix} \quad (10)$$

where $\mathbf{G}_{\alpha kl}^{(1)} \in \mathbb{C}^{d_{\alpha k} \times d_{\beta l}}$, $\mathbf{G}_{\alpha kl}^{(2)} \in \mathbb{C}^{d_{\alpha k} \times (N_{\beta l} - d_{\beta l})}$, $\mathbf{G}_{\alpha kl}^{(3)} \in \mathbb{C}^{(N_{\alpha k} - d_{\alpha k}) \times d_{\beta l}}$, and $\mathbf{G}_{\alpha kl}^{(4)} \in \mathbb{C}^{(N_{\alpha k} - d_{\alpha k}) \times (N_{\beta l} - d_{\beta l})}$. The following theorem establishes a sufficient condition on the IA feasibility.

Theorem 2 (Sufficient condition): For the MIMO R-TDD cellular network, $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$ is feasible almost surely if (9a) to (9c) are satisfied and (11) is a full row rank matrix, where

$$\begin{aligned} \mathbf{G}'_{\alpha kl} &= \text{diag}[d_{\alpha k}] \left(\mathbf{G}_{\alpha kl}^{(3)T} \right), \\ \mathbf{G}''_{\alpha kl} &= \begin{bmatrix} \text{diag}[d_{\beta l}] \left(\mathbf{G}_{\alpha kl}^{(2)} [1] \right) \\ \text{diag}[d_{\beta l}] \left(\mathbf{G}_{\alpha kl}^{(2)} [2] \right) \\ \vdots \\ \text{diag}[d_{\beta l}] \left(\mathbf{G}_{\alpha kl}^{(2)} [d_{\alpha k}] \right) \end{bmatrix} \end{aligned}$$

and the definitions of $\mathbf{G}_{\alpha kl}^{(2)}$ and $\mathbf{G}_{\alpha kl}^{(3)}$ are given by (10).

Proof: We refer to Section IV-B for the proof. ■

Depending on the network configuration, there may exist a DoF gap between the necessary condition and the sufficient condition stated in Theorems 1 and 2 respectively. The following two examples show one case where the sum DoF satisfying the necessary condition and the sufficient condition are the same, thereby it is optimal from Definition 2, and the other case where there exist a DoF gap. For both cases, more importantly, the proposed one-shot linear IA strictly enlarges the sum DoF compared with the sum DoF achievable by operating one of the two cells, which is given by

$$d_{\text{sum, single}} = \max \left\{ \min \left(M_{\alpha}, \sum_{k=1}^K N_{\alpha k} \right), \min \left(M_{\beta}, \sum_{l=1}^L N_{\beta l} \right) \right\}. \quad (12)$$

Example 1 (An example network where its optimal d_{sum} is characterized by Theorems 1 and 2): Consider the $(10, (4, 6, 6)) \times (13, (3, 6))$ MIMO R-TDD cellular network. For this configuration, Theorem 1 implies $d_{\text{sum}} \leq 13$, which is feasible from Theorem 2. Therefore, the optimal sum DoF is given by $d_{\text{sum}} = 13$. However, the single-cell lower bound in (12) only achieves $d_{\text{sum, single}} = 10$.

Example 2 (An example network where there is a DoF from Theorems 1 and 2): Consider the $(8, (2, 3, 8)) \times (12, (3, 7))$ MIMO R-TDD cellular network. For this configuration, Theorem 1 implies $d_{\text{sum}} \leq 12$ but $d_{\text{sum}} = 11$ is feasible from Theorem 2, which shows the sum DoF gap of one. Although there exists the DoF gap from its upper bound, the proposed IA in Theorem 2 strictly improves the sum DoF compared with the single-cell lower bound, given by $d_{\text{sum, single}} = 10$.

$$\mathbf{G}_\alpha = \begin{bmatrix} \mathbf{G}'_{\alpha 11} & 0 & \cdots & 0 & \cdots & 0 & \mathbf{G}''_{\alpha 11} & 0 & \cdots & 0 \\ \mathbf{G}'_{\alpha 12} & 0 & \cdots & 0 & \cdots & 0 & 0 & \mathbf{G}''_{\alpha 12} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}'_{\alpha 1L} & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & \mathbf{G}''_{\alpha 1L} \\ 0 & \mathbf{G}'_{\alpha 21} & \cdots & 0 & \cdots & 0 & \mathbf{G}''_{\alpha 21} & 0 & \cdots & 0 \\ 0 & \mathbf{G}'_{\alpha 22} & \ddots & 0 & \cdots & 0 & 0 & \mathbf{G}''_{\alpha 22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \mathbf{G}'_{\alpha 2L} & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & \mathbf{G}''_{\alpha 2L} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \mathbf{G}'_{\alpha K1} & \mathbf{G}''_{\alpha K1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & \mathbf{G}'_{\alpha K2} & 0 & \mathbf{G}''_{\alpha K2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \mathbf{G}'_{\alpha KL} & 0 & 0 & \cdots & \mathbf{G}''_{\alpha KL} \end{bmatrix} \quad (11)$$

Remark 2 (Duality for the IA feasibility): Suppose that a DoF tuple $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$ satisfies the set of conditions in Theorem 2, meaning that it is feasible, for the $(M_\alpha, (N_{\alpha 1}, \dots, N_{\alpha K})) \times (M_\beta, (N_{\beta 1}, \dots, N_{\beta L}))$ MIMO R-TDD cellular network. Then $(d_{\beta 1}, \dots, d_{\beta L}, d_{\alpha 1}, \dots, d_{\alpha K})$ is feasible for its dual MIMO R-TDD cellular network, i.e., the $(M_\beta, (N_{\beta 1}, \dots, N_{\beta L})) \times (M_\alpha, (N_{\alpha 1}, \dots, N_{\alpha K}))$ MIMO R-TDD cellular network.

Example 3 (The dual network of Example 1): From Example 1, the optimal sum DoF of the $(13, (3, 6)) \times (10, (4, 6, 6))$ MIMO R-TDD cellular network is also given by $d_{\text{sum}} = 13$.

In many cases of interest, each user may require the same DoF, i.e., $d_{\alpha k} = d_\alpha, d_{\beta l} = d_\beta$ for all $k \in [1 : K]$ and $l \in [1 : L]$. By focusing on such a symmetric DoF, we establish a sufficient condition with a more explicit expression than Theorem 2.

Theorem 3 (Sufficient condition for symmetric DoF):

For the MIMO R-TDD cellular network, a symmetric DoF d_α and d_β , i.e., $d_{\alpha k} = d_\alpha, d_{\beta l} = d_\beta$ for all $k \in [1 : K]$ and $l \in [1 : L]$, is feasible almost surely if the following set of conditions are satisfied:

$$Kd_\alpha \leq M_\alpha, \quad (13a)$$

$$Ld_\beta \leq M_\beta, \quad (13b)$$

$$Kd_\alpha + Ld_\beta \leq \max(M_\alpha, M_\beta), \quad (13c)$$

$$\text{mod}(N_{\alpha k} - d_\alpha, d_\beta) = \text{mod}(N_{\beta l} - d_\beta, d_\alpha) = 0, \quad \forall k, l, \quad (13d)$$

$$\begin{aligned} |\mathcal{I}_\alpha| |\mathcal{I}_\beta| d_\alpha d_\beta &\leq \sum_{k \in \mathcal{I}_\alpha} d_\alpha (N_{\alpha k} - d_\alpha) \\ &\quad + \sum_{l \in \mathcal{I}_\beta} d_\beta (N_{\beta l} - d_\beta), \quad \forall \mathcal{I}_\alpha, \mathcal{I}_\beta \end{aligned} \quad (13e)$$

where $k \in [1 : K], l \in [1 : L], \mathcal{I}_\alpha \subseteq [1 : K]$, and $\mathcal{I}_\beta \subseteq [1 : L]$.

Proof: See Section IV-C for the proof. ■

Remark 3 (Necessary and sufficient condition for divisible symmetric DoF): For symmetric DoF, the conditions (9a), (9b), (9c), and (9e) in Theorem 1 yield the conditions (13a), (13b),

(13c), and (13e) in Theorem 3. Therefore, if (13d) is satisfied for given d_α and d_β , then Theorem 3 provides the necessary and sufficient condition, i.e., the set of conditions (13a), (13b), (13c), and (13e).

Example 4 (An example network with symmetric DoF): Consider the $(12, (6, 6, 8)) \times (16, (6, 6))$ MIMO R-TDD cellular network. Then $d_\alpha = 4$ and $d_\beta = 2$ are feasible from Theorem 3, which achieves $d_{\text{sum}} = 16$. Theorem 1 implies $d_{\text{sum}} \leq 16$ for this configuration, hence, it is the optimal sum DoF and provides a larger sum DoF than $d_{\text{sum, single}} = 12$.

IV. IA FEASIBILITY FOR MIMO R-TDD CELLULAR NETWORKS

In this section, we prove Theorems 1, 2, and 3 stated in Section III.

A. Proof of Theorem 1

In order to prove Theorem 1, we first introduce the sum DoF of the two-user MIMO IC in [31]. For the two-user MIMO IC with M_1 transmit antennas and N_1 received antennas for the first transmission pair and M_2 transmit antennas and N_2 received antennas for the second transmission pair, the optimal sum DoF is given by

$$\min \{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\}. \quad (14)$$

From (14), for given $\mathcal{I}_\alpha \subseteq [1 : K]$ and $\mathcal{I}_\beta \subseteq [1 : L]$,

$$\begin{aligned} &\sum_{k \in \mathcal{I}_\alpha} d_{\alpha k} + \sum_{l \in \mathcal{I}_\beta} d_{\beta l} \\ &\leq \min \left\{ M_\alpha + \sum_{l \in \mathcal{I}_\beta} N_{\beta l}, M_\beta + \sum_{k \in \mathcal{I}_\alpha} N_{\alpha k}, \right. \\ &\quad \left. \max(M_\alpha, M_\beta), \max \left(\sum_{k \in \mathcal{I}_\alpha} N_{\alpha k}, \sum_{l \in \mathcal{I}_\beta} N_{\beta l} \right) \right\} \end{aligned} \quad (15)$$

which corresponds to the sum DoF if full cooperation is allowed among the users in \mathcal{I}_α and the users in \mathcal{I}_β respectively.

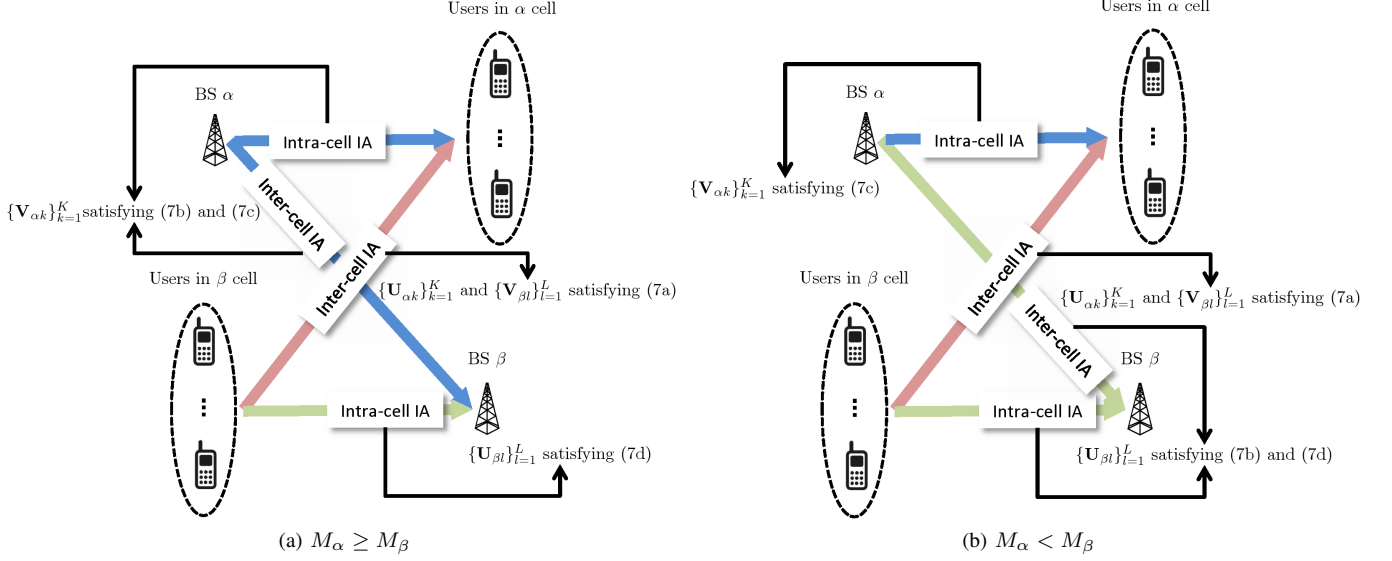


Fig. 2. The relations of the precoding, postcoding matrices and its interferences.

Hence (15) must be satisfied for all $\mathcal{I}_\alpha \subseteq [1 : K]$ and $\mathcal{I}_\beta \subseteq [1 : L]$, which yields the conditions (9a) to (9d).

The last condition (9e) is derived in a similar manner in [35]. From the IA feasibility conditions (7e) and (7f), $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ should be full column rank matrices. Therefore, we restrict $\mathbf{U}_{\alpha k}$ and $\mathbf{V}_{\beta l}$ such that $\text{rank}(\mathbf{U}_{\alpha k}) = d_{\alpha k}$ and $\text{rank}(\mathbf{V}_{\beta l}) = d_{\beta l}$ for all $k \in [1 : K]$ and $l \in [1 : L]$. Then they can be rewritten as

$$\mathbf{U}_{\alpha k} = \begin{bmatrix} \mathbf{I}_{d_{\alpha k}} \\ \bar{\mathbf{U}}_{\alpha k} \end{bmatrix} \mathbf{A}_{\alpha k}^{-1}, \mathbf{V}_{\beta l} = \begin{bmatrix} \mathbf{I}_{d_{\beta l}} \\ \bar{\mathbf{V}}_{\beta l} \end{bmatrix} \mathbf{A}_{\beta l}^{-1} \quad (16)$$

where $\bar{\mathbf{U}}_{\alpha k} \in \mathbb{C}^{(N_{\alpha k} - d_{\alpha k}) \times d_{\alpha k}}$ and $\bar{\mathbf{V}}_{\beta l} \in \mathbb{C}^{(N_{\beta l} - d_{\beta l}) \times d_{\beta l}}$ are some arbitrary matrices and $\mathbf{A}_{\alpha k} \in \mathbb{C}^{N_{\alpha k} \times N_{\alpha k}}$ and $\mathbf{A}_{\beta l} \in \mathbb{C}^{N_{\beta l} \times N_{\beta l}}$ are some invertible matrices.

From (10) and (16), the IA feasibility condition (7a) is given as

$$\begin{bmatrix} \mathbf{I}_{d_{\alpha k}} & \bar{\mathbf{U}}_{\alpha k}^\dagger \end{bmatrix} \begin{bmatrix} \mathbf{G}_{\alpha kl}^{(1)} & \mathbf{G}_{\alpha kl}^{(2)} \\ \mathbf{G}_{\alpha kl}^{(3)} & \mathbf{G}_{\alpha kl}^{(4)} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{d_{\beta l}} \\ \bar{\mathbf{V}}_{\beta l} \end{bmatrix} = \mathbf{0} \quad (17)$$

for all $k \in [1 : K]$ and $l \in [1 : L]$. Let us now consider fixed \mathcal{I}_α and \mathcal{I}_β , where $\mathcal{I}_\alpha \subseteq [1 : K]$ and $\mathcal{I}_\beta \subseteq [1 : L]$. By treating the elements of $\bar{\mathbf{U}}_{\alpha k}$ and $\bar{\mathbf{V}}_{\beta l}$ as controllable variables and considering a set of conditions for all $k \in \mathcal{I}_\alpha$ and $l \in \mathcal{I}_\beta$ in (17), we have

$$\sum_{k \in \mathcal{I}_\alpha} \sum_{l \in \mathcal{I}_\beta} d_{\alpha k} d_{\beta l} \leq \sum_{k \in \mathcal{I}_\alpha} d_{\alpha k} (N_{\alpha k} - d_{\alpha k}) + \sum_{l \in \mathcal{I}_\beta} d_{\beta l} (N_{\beta l} - d_{\beta l})$$

where the left-hand side corresponds to the number of equations and the right-hand side corresponds to the number of variables. Since the above inequality should be satisfied for all $\mathcal{I}_\alpha \subseteq [1 : K]$ and $\mathcal{I}_\beta \subseteq [1 : L]$, we have the condition (9e). In conclusion, Theorem 1 holds.

B. Proof of Theorem 2

In this subsection, we prove Theorem 2. We show that precoding and postcoding matrices satisfying the IA feasibility conditions in Definition 1 exist almost surely if the set of conditions in Theorem 2 are satisfied. The overall construction of precoding and postcoding matrices is as follows:

- Step 1: Construct $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) as shown in both Fig. 2a and Fig. 2b.
- Step 2: For $M_\alpha \geq M_\beta$, first construct $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7d) and then construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ satisfying (7b) and (7c) as shown in Fig. 2a. On the other hand, for $M_\alpha < M_\beta$, first construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ satisfying (7c) and then construct $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7b) and (7d) as shown in Fig. 2b.

In the following, we show that $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ in Step 1 can be established almost surely if \mathbf{G}_α in (11) is a full row rank matrix and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ and $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ in Step 2 can be established almost surely if the conditions in (9a) to (9c) are satisfied. In Section V, we further explain in details how to construct such $\{\mathbf{U}_{\alpha k}, \mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}, \mathbf{V}_{\beta l}\}_{l=1}^L$. Especially, $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ are constructed by using the iterative algorithm proposed in [53].

Remark 4 (Inter-cell IA from BS α to BS β): For the above construction, inter-cell IA is applied at BS α using precoding matrices in order to satisfy (7b) and then BS β decodes its streams using postcoding matrices as described in Fig. 2a if $M_\alpha \geq M_\beta$. On the other hand, BS β decodes its streams without inter-cell IA at BS α as described in Fig. 2b if $M_\alpha < M_\beta$. It has been shown in [52] that this approach achieves the optimal sum DoF for the case where each user has a single antenna.

1) *Construction of $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$:* We first concentrate on constructing $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ that satisfies (7a). Since these matrices should also satisfy (7e) and

(7f), we assume that

$$\mathbf{U}_{\alpha k} = \begin{bmatrix} \mathbf{I}_{d_{\alpha k}} \\ \bar{\mathbf{U}}_{\alpha k} \end{bmatrix}, \mathbf{V}_{\beta l} = \begin{bmatrix} \mathbf{I}_{d_{\beta l}} \\ \bar{\mathbf{V}}_{\beta l} \end{bmatrix}. \quad (18)$$

That is, we only control $\bar{\mathbf{U}}_{\alpha k} \in \mathbb{C}^{(N_{\alpha k} - d_{\alpha k}) \times d_{\alpha k}}$ and $\bar{\mathbf{V}}_{\beta l} \in \mathbb{C}^{(N_{\beta l} - d_{\beta l}) \times d_{\beta l}}$ in order to satisfy (7a). Let

$$q(k, l, m, n) = \sum_{k'=1}^{k-1} \sum_{l'=1}^l d_{\alpha k'} d_{\beta l'} + \sum_{l'=1}^l d_{\alpha k} d_{\beta l'} + (m-1) d_{\beta l} + n$$

for $m \in [1 : d_{\alpha k}]$ and $n \in [1 : d_{\beta l}]$. Then define $r_{q(k, l, m, n)}$ as the (m, n) th element of $\mathbf{U}_{\alpha k}^\dagger \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l}$, that is given by

$$\begin{aligned} r_{q(k, l, m, n)} &= \mathbf{G}_{\alpha k l}^{(1)}[m, n] + \sum_{i=1}^{N_{\beta l} - d_{\beta l}} \mathbf{G}_{\alpha k l}^{(2)}[m, i] \bar{\mathbf{V}}_{\beta l}[i, n] \\ &\quad + \sum_{j=1}^{N_{\alpha k} - d_{\alpha k}} \mathbf{G}_{\alpha k l}^{(3)}[j, m] \bar{\mathbf{U}}_{\alpha k}^\dagger[j, n] \\ &\quad + \sum_{j=1}^{N_{\alpha k} - d_{\alpha k}} \sum_{i=1}^{N_{\beta l} - d_{\beta l}} \mathbf{G}_{\alpha k l}^{(4)}[j, i] \bar{\mathbf{U}}_{\alpha k}^\dagger[j, n] \bar{\mathbf{V}}_{\beta l}[i, n] \\ &= \mathbf{G}_{\alpha k l}^{(1)}[m, n] + \mathbf{G}_{\alpha} [q(k, l, m, n)] \mathbf{f}_1 \\ &\quad + \sum_{j=1}^{N_{\alpha k} - d_{\alpha k}} \sum_{i=1}^{N_{\beta l} - d_{\beta l}} \mathbf{G}_{\alpha k l}^{(4)}[j, i] \bar{\mathbf{U}}_{\alpha k}^\dagger[j, n] \bar{\mathbf{V}}_{\beta l}[i, n] \end{aligned} \quad (19)$$

where

$$\mathbf{f}_1 = \begin{bmatrix} \text{vec}(\bar{\mathbf{U}}_{\alpha 1}^\dagger) \\ \vdots \\ \text{vec}(\bar{\mathbf{U}}_{\alpha K}^\dagger) \\ \text{vec}(\bar{\mathbf{V}}_{\beta 1}) \\ \vdots \\ \text{vec}(\bar{\mathbf{V}}_{\beta L}) \end{bmatrix}. \quad (20)$$

Hence, from (19), (7a) can be expressed as

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{\sum_{k'=1}^K \sum_{l'=1}^L d_{\alpha k'} d_{\beta l'}} \end{bmatrix} = \mathbf{f}_0 + \mathbf{G}_{\alpha} \mathbf{f}_1 + \mathbf{f}_2 = \mathbf{0} \quad (21)$$

where \mathbf{f}_0 is the zero-order polynomial vector, independent of $\{\bar{\mathbf{U}}_{\alpha k}\}_{k=1}^K$ and $\{\bar{\mathbf{V}}_{\beta l}\}_{l=1}^L$, and $\mathbf{G}_{\alpha} \mathbf{f}_1$ and \mathbf{f}_2 are respectively the first-order and the second-order polynomial vectors with respect to $\{\bar{\mathbf{U}}_{\alpha k}\}_{k=1}^K$ and $\{\bar{\mathbf{V}}_{\beta l}\}_{l=1}^L$. The definition of \mathbf{G}_{α} is given by (11).

In order to show the existence of $\{\bar{\mathbf{U}}_{\alpha k}\}_{k=1}^K$ and $\{\bar{\mathbf{V}}_{\beta l}\}_{l=1}^L$ satisfying (21), we introduce the following algebraic geometric lemma in [36], which has been used for the feasibility problem of the MIMO K -user IC.

Lemma 1 (Linear independence leads to a solution): Consider a set of R polynomials represented by $\{g_i \in \mathbb{C}[x_1, x_2, \dots, x_S]\}_{i=1}^R$, where $\mathbb{C}[x_1, x_2, \dots, x_S]$ denotes an algebraically closed field of rational functions in variables x_1, x_2, \dots, x_S with coefficients drawn from \mathbb{C} , $g_i =$

$\sum_{j=1}^S f_{ij} x_j + e_i$, and e_i is a set of polynomials with degrees no less than two. If the coefficient vectors $\mathbf{g}_i = [f_{i1} \ f_{i2} \ \dots \ f_{iS}]_{i=1}^R$ are linearly independent, then the polynomials $\{g_i\}_{i=1}^R$ are algebraically independent and, as a result, the linear equations $\{g_i = c_i\}_{i=1}^R$ have a solution almost surely, where the elements of $\{c_i \in \mathbb{C}\}_{i=1}^R$ are independently drawn from continuous distributions.

Notice that the above lemma can be applied to (21) since the elements of \mathbf{f}_0 are independently drawn from continuous distributions. Therefore, $\{\bar{\mathbf{U}}_{\alpha k}\}_{k=1}^K$ and $\{\bar{\mathbf{V}}_{\beta l}\}_{l=1}^L$ satisfying (21) exist almost surely if the row vectors of \mathbf{G}_{α} are linearly independent. In conclusion, $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) exist almost surely if \mathbf{G}_{α} is a full row rank matrix.

2) *Construction of $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$:* Let us now consider the construction of $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$, for given $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a).

First consider the case where $M_{\alpha} \geq M_{\beta}$. For this case, the condition (9a) becomes inactive from the condition (9c). We construct $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ only satisfying (7d). Then there exists a nonzero $\mathbf{U}_{\beta l}$ satisfying (7d) almost surely if

$$d_{\beta l} \leq M_{\beta} - \sum_{l'=1, l' \neq l}^L d_{\beta l'}$$

since the total number of intra-cell interference dimensions is given by $\sum_{l'=1, l' \neq l}^L d_{\beta l'}$, which yields the condition (9b). Then we construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ satisfying (7b) and (7c) for given $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$. Then there exists a nonzero $\mathbf{V}_{\alpha k}$ satisfying (7b) and (7c) almost surely if

$$d_{\alpha k} \leq M_{\alpha} - \sum_{k'=1, k' \neq k}^K d_{\alpha k'} - \sum_{l'=1}^L d_{\beta l'}$$

since the total number of intra-cell and inter-cell interference dimensions is given by $\sum_{k'=1, k' \neq k}^K d_{\alpha k'} + \sum_{l'=1}^L d_{\beta l'}$, which yields the condition (9c) because of $M_{\alpha} \geq M_{\beta}$.

Now consider the case where $M_{\alpha} < M_{\beta}$. For this case, (9b) becomes inactive from (9c). We construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ only satisfying (7c). Then there exists a nonzero $\mathbf{V}_{\alpha k}$ satisfying (7c) almost surely if

$$d_{\alpha k} \leq M_{\alpha} - \sum_{k'=1, k' \neq k}^K d_{\alpha k'},$$

which yields (9a). Then we construct $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7b) and (7d) for given $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$. Then there exists a nonzero $\mathbf{U}_{\beta l}$ satisfying (7b) and (7d) almost surely if

$$d_{\beta l} \leq M_{\beta} - \sum_{l'=1, l' \neq l}^L d_{\beta l'} - \sum_{k'=1}^K d_{\alpha k'},$$

which yields the condition (9c) because of $M_{\alpha} < M_{\beta}$.

In conclusion, for both cases, $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7b) to (7d) exist almost surely if (9a) to (9c) are satisfied.

3) *Linear independence for the desired streams:* From the above construction, we can easily show that $\mathbf{U}_{\alpha k}$ and $\mathbf{V}_{\beta l}$ are not function of $\mathbf{H}_{\alpha k}$ and $\mathbf{H}_{\beta l}$ for all $k \in [1 : K]$ and $l \in [1 : L]$. Therefore, the conditions (7e) and (7f) are satisfied almost surely. In conclusion, Theorem 2 holds.

C. Proof of Theorem 3

In this subsection, we prove Theorem 3. We construct $\{\mathbf{U}_{\alpha k}, \mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}, \mathbf{V}_{\beta l}\}_{l=1}^L$ in the same manner used in the proof of Theorem 2.

First, let us investigate the existence of $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a), which is rewritten as (17) from (18). Then denote the left-hand side of (17) by \mathbf{F}_{kl} that is given by

$$\mathbf{F}_{kl} = \mathbf{G}_{\alpha kl}^{(1)} + \mathbf{G}_{\alpha kl}^{(2)} \bar{\mathbf{V}}_{\beta l} + \bar{\mathbf{U}}_{\alpha k}^\dagger \mathbf{G}_{\alpha kl}^{(3)} + \bar{\mathbf{U}}_{\alpha k}^\dagger \mathbf{G}_{\alpha kl}^{(4)} \bar{\mathbf{V}}_{\beta l}. \quad (22)$$

The Jacobian matrix of (22) with respect to variables $\{\bar{\mathbf{U}}_{\alpha k}\}_{k=1}^K$ and $\{\bar{\mathbf{V}}_{\beta l}\}_{l=1}^L$ is defined as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \text{vec}(\mathbf{F})}{\partial \text{vec}(\bar{\mathbf{U}}, \bar{\mathbf{V}})} \end{bmatrix} \quad (23)$$

where $\text{vec}(\mathbf{F}) = [\text{vec}(\mathbf{F}_{11})^T, \dots, \text{vec}(\mathbf{F}_{KL})^T]^T$ and

$$\text{vec}(\bar{\mathbf{U}}, \bar{\mathbf{V}}) = [\text{vec}(\bar{\mathbf{U}}_{\alpha 1}^\dagger)^T, \dots, \text{vec}(\bar{\mathbf{U}}_{\alpha K}^\dagger)^T, \\ \text{vec}(\bar{\mathbf{V}}_{\beta 1})^T, \dots, \text{vec}(\bar{\mathbf{V}}_{\beta L})^T]^T.$$

We then introduce the following lemma, which will be used for showing the existence of $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) almost surely.

Lemma 2 (Existence of a specific channel realization): For the MIMO R-TDD cellular network, if (13d) and (13e) are satisfied, then there exist $\{\mathbf{G}_{\alpha kl}^{(i)}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$, denoted by $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$, such that \mathbf{J} is non-singular for given $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$. Furthermore, there exist $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) for given $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$.

Proof: Assume that the symmetric DoF d_α and d_β satisfy (13d) and (13e). First we explain how to establish $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$ that guarantees non-singularity of \mathbf{J} . Set $\mathbf{G}_{\alpha kl}^{(1)*} = \mathbf{0}$ and $\mathbf{G}_{\alpha kl}^{(4)*} = \mathbf{0}$ for all k and l . Then from (13d) and (22)

$$\mathbf{F}_{kl} = \begin{bmatrix} \bar{\mathbf{U}}_{\alpha k,1}^\dagger & \bar{\mathbf{U}}_{\alpha k,2}^\dagger & \cdots & \bar{\mathbf{U}}_{\alpha k,A_k}^\dagger \end{bmatrix} \begin{bmatrix} \mathbf{G}_{\alpha kl,1}^{(3)} \\ \mathbf{G}_{\alpha kl,2}^{(3)} \\ \vdots \\ \mathbf{G}_{\alpha kl,A_k}^{(3)} \end{bmatrix} \\ + \begin{bmatrix} \mathbf{G}_{\alpha kl,1}^{(2)} \\ \mathbf{G}_{\alpha kl,2}^{(2)} \\ \vdots \\ \mathbf{G}_{\alpha kl,B_l}^{(2)} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_{\beta l,1} & \bar{\mathbf{V}}_{\beta l,2} & \cdots & \bar{\mathbf{V}}_{\beta l,B_l} \end{bmatrix}, \quad (24)$$

where $\bar{\mathbf{U}}_{\alpha k,i} \in \mathbb{C}^{d_\beta \times d_\alpha}$, $\bar{\mathbf{V}}_{\beta l,j} \in \mathbb{C}^{d_\alpha \times d_\beta}$, $\mathbf{G}_{\alpha kl,i}^{(3)} \in \mathbb{C}^{d_\beta \times d_\beta}$, $\mathbf{G}_{\alpha kl,j}^{(2)} \in \mathbb{C}^{d_\alpha \times d_\alpha}$ and $i \in [1:A_k]$, $j \in [1:B_l]$, $A_k = (N_{\alpha k} - d_\alpha)/d_\beta$, $B_l = (N_{\beta l} - d_\beta)/d_\alpha$.

We now introduce the bipartite graph $G(\mathcal{X}, \mathcal{Y}, \mathcal{E})$ depicted in Fig. 3. The vertex set \mathcal{X} consists of \mathbf{F}_{kl} for all $k \in [1:K]$, $l \in [1:L]$ and the vertex set \mathcal{Y} consists of $\bar{\mathbf{U}}_{\alpha k,i}$, $\bar{\mathbf{V}}_{\beta l,j}$ for all $i \in [1:A_k]$, $j \in [1:B_l]$, $k \in [1:K]$, $l \in [1:L]$. There exist an edge between $\bar{\mathbf{U}}_{\alpha k,i}$ (or $\bar{\mathbf{V}}_{\beta l,j}$) and \mathbf{F}_{kl} if $\bar{\mathbf{U}}_{\alpha k,i}$ (or $\bar{\mathbf{V}}_{\beta l,j}$) appears in the definition of \mathbf{F}_{kl} in (24) where the

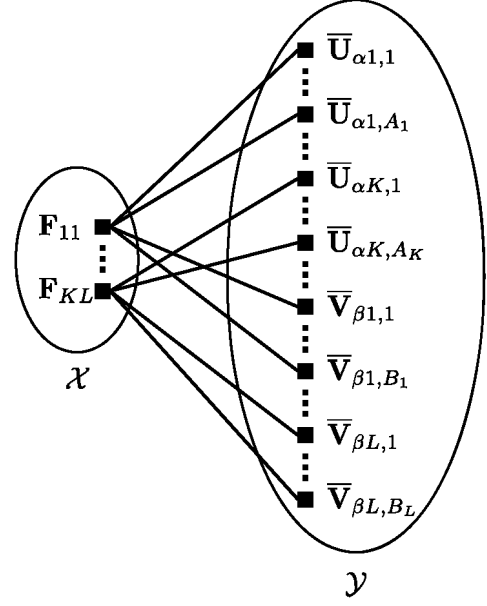


Fig. 3. The bipartite graph of Lemma 2.

edge set is denoted by \mathcal{E} . From Hall's marriage theorem [54], there is a complete matching in $G(\mathcal{X}, \mathcal{Y}, \mathcal{E})$ if and only if $|\mathcal{X}_s| \leq |N_G(\mathcal{X}_s)|$ for all possible subsets of vertices $\mathcal{X}_s \subseteq \mathcal{X}$, where $N_G(\mathcal{X}_s)$ denotes the set of neighbors of \mathcal{X}_s . Notice that the above condition is satisfied by (13e), which guarantees the existence of a complete matching in $G(\mathcal{X}, \mathcal{Y}, \mathcal{E})$.

Suppose that $\mathcal{Y}_c \subseteq \mathcal{Y}$ denotes the set of vertices included in a complete matching. Then set $\mathbf{G}_{\alpha kl,i}^{(3)*} = \mathbf{I}_{d_\beta}$ if $\bar{\mathbf{U}}_{\alpha k,i}$ is included in \mathcal{Y}_c , otherwise set $\mathbf{G}_{\alpha kl,i}^{(3)*} = \mathbf{0}$ for all $i \in [1:A_k]$ and $k \in [1:K]$. Similarly set $\mathbf{G}_{\alpha kl,j}^{(2)*} = \mathbf{I}_{d_\alpha}$ if $\bar{\mathbf{V}}_{\beta l,j}$ is included in \mathcal{Y}_c , otherwise set $\mathbf{G}_{\alpha kl,j}^{(2)*} = \mathbf{0}$ for all $j \in [1:B_l]$ and $l \in [1:L]$.

As a result, \mathbf{J} in (23) becomes a block permutation matrix for given $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$, which is non-singular. Moreover, setting $\bar{\mathbf{U}}_{\alpha k} = \mathbf{0}$ for all $k \in [1:K]$ and $\bar{\mathbf{V}}_{\beta l} = \mathbf{0}$ for all $l \in [1:L]$ satisfies (7a) for given $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$. ■

By the same analysis in [35], [37], [44], if there exists a specific set of $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$ that guarantees non-singular \mathbf{J} and the existence of $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a), then there exist $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) almost surely for a set of generic channel matrices $\{\mathbf{G}_{\alpha kl}^{(i)}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$. From Lemma 2, we can find such $\{\mathbf{G}_{\alpha kl}^{(i)*}\}_{i \in [1:4], k \in [1:K], l \in [1:L]}$ if (13d) and (13e) are satisfied. In conclusion, if (13d) and (13e) are satisfied, $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a) exist almost surely.

Proving the existence of $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ and $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ satisfying (7b) to (7d) is the same as that in Theorem 2, which yields the conditions (13a) to (13c). Therefore, the conditions (7e) and (7f) are satisfied almost surely from the same manner as Theorem 2. Therefore, Theorem 3 holds.

V. ITERATIVE CONSTRUCTION OF PRECODING AND POSTCODING MATRICES

Up to now, we considered necessary or sufficient conditions on the IA feasibility for MIMO R-TDD cellular networks. In this section, we focus on how to construct precoding and postcoding matrices assuming that $(d_{\alpha 1}, \dots, d_{\alpha K}, d_{\beta 1}, \dots, d_{\beta L})$ satisfies the sufficient condition in Theorem 2. The overall procedure follows the same steps in Section IV-B. Specifically, we apply the iterative method proposed in [53] for constructing $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a), i.e., Step 1. Then we apply pseudo-inverse zero-forcing for constructing $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7b) to (7d), i.e., Step 2.

We assume that the transmit power of $P_{\alpha k}$, $k \in [1 : K]$, is used for user (α, k) and the transmit power of $P_{\beta l}$, $l \in [1 : L]$, is used for user (β, l) . Hence, the transmit power of BS α is given by $P_\alpha = \sum_{k=1}^K P_{\alpha k}$. We further assume equal power allocation between multiple streams for each user, i.e., $\frac{P_{\alpha k}}{d_{\alpha k}}$ is allocated for each of $d_{\alpha k}$ streams for user (α, k) and $\frac{P_{\beta l}}{d_{\beta l}}$ is allocated for each of $d_{\beta l}$ streams for user (β, l) . In the following, we describe the proposed construction based on the normalized precoding and postcoding matrices. That is, the squared norm of each column vector of precoding and postcoding matrices is normalized by one.

A. Construction of $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$

As mentioned above, we apply the iterative method in [53] for constructing $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a). To describe the proposed construction, we define $\{\mathbf{U}_{\alpha k}^{[0]} \in \mathbb{C}^{N_{\alpha k} \times d_{\alpha k}}\}_{k=1}^K$, each of which is randomly chosen from the set of unitary matrices. Let $\{\mathbf{U}_{\alpha k}^{[i]}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}^{[i]}\}_{l=1}^L$ denote the precoding and postcoding matrices at the i th iteration respectively. For the i th iteration, we first update $\mathbf{V}_{\beta l}^{[i]}$ from $\mathbf{U}_{\alpha k}^{[i-1]}$ and then sequentially update $\mathbf{U}_{\alpha k}^{[i]}$ from $\mathbf{V}_{\beta l}^{[i]}$. In the following, we state in details how to update $\mathbf{V}_{\beta l}^{[i]}$ and $\mathbf{U}_{\alpha k}^{[i]}$.

In order to update $\mathbf{V}_{\beta l}^{[i]}$ from $\mathbf{U}_{\alpha k}^{[i-1]}$, calculate the inter-cell interference covariance matrix for user (β, l) defined by

$$\mathbf{C}_{\beta l}^{[i]} = \sum_{k=1}^K \frac{P_{\alpha k}}{d_{\alpha k}} \mathbf{G}_{\alpha k l}^\dagger \mathbf{U}_{\alpha k}^{[i-1]} \left(\mathbf{G}_{\alpha k l}^\dagger \mathbf{U}_{\alpha k}^{[i-1]} \right)^\dagger. \quad (25)$$

Then set

$$\mathbf{V}_{\beta l}^{[i]}[:, p] = \Gamma \left[\mathbf{C}_{\beta l}^{[i]}, p \right], \quad \forall p \in [1 : d_{\beta l}] \quad (26)$$

where $\Gamma[\mathbf{A}, n]$ denotes the normalized eigenvector corresponding to the n th smallest eigenvalue of \mathbf{A} and $\mathbf{A}[:, m]$ denotes the m th column vector of \mathbf{A} . Hence the resulting $\mathbf{V}_{\beta l}^{[i]}$ chooses the signal subspace that contains the least interference power.

Similarly, to update $\mathbf{U}_{\alpha k}^{[i]}$ from $\mathbf{V}_{\beta l}^{[i]}$, calculate the inter-cell interference covariance matrix at user (α, k) defined by

$$\mathbf{C}_{\alpha k}^{[i]} = \sum_{l=1}^L \frac{P_{\beta l}}{d_{\beta l}} \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l}^{[i]} \left(\mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l}^{[i]} \right)^\dagger \quad (27)$$

and set

$$\mathbf{U}_{\alpha k}^{[i]}[:, q] = \Gamma \left[\mathbf{C}_{\alpha k}^{[i]}, q \right], \quad \forall q \in [1 : d_{\alpha k}]. \quad (28)$$

TABLE I
ITERATIVE CONSTRUCTION FOR $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ AND $\{\mathbf{V}_{\beta l}\}_{l=1}^L$.

1: Initialization: Fix $\lambda \in \mathbb{N}$ and set $\{\mathbf{U}_{\alpha k}^{[0]}\}_{k=1}^K$, each of which is chosen from the set of unitary matrices uniformly at random.
2: for $i \in [1 : \lambda]$ do
3: Calculate $\mathbf{C}_{\beta l}^{[i]}$ defined in (25) and update $\mathbf{V}_{\beta l}^{[i]}$ as in (26).
4: Calculate $\mathbf{C}_{\alpha k}^{[i]}$ defined in (27) and update $\mathbf{U}_{\alpha k}^{[i]}$ as in (28).
5: end for
6: Result: Set $\mathbf{U}_{\alpha k} = \mathbf{U}_{\alpha k}^{[\lambda]}$ for all $k \in [1 : K]$ and $\mathbf{V}_{\beta l} = \mathbf{V}_{\beta l}^{[\lambda]}$ for all $l \in [1 : L]$.

Table I summarizes the proposed iterative construction.

After the λ th iteration, the leakage interference power for user (α, k) is given by

$$I_{\alpha k}^{[\lambda]} = \text{tr} \left(\mathbf{U}_{\alpha k}^{[\lambda] \dagger} \mathbf{C}_{\alpha k}^{[\lambda]} \mathbf{U}_{\alpha k}^{[\lambda]} \right)$$

where $\text{tr}(\cdot)$ denotes the trace operation. The total leakage interference power after the λ th iteration is then given by $I^{[\lambda]} = \sum_{k=1}^K I_{\alpha k}^{[\lambda]}$. It was shown in [53] that the total leakage interference power converges to zero as the number iterations increases, i.e., $I^{[\lambda]} \rightarrow 0$ as $\lambda \rightarrow \infty$. Hence we are able to decrease $I^{[\lambda]}$ as an arbitrarily small value for large enough λ , see also Fig. 4.

B. Construction of $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$

We now state in details how to construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ for given $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ satisfying (7a). That is, we assume that $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ are constructed by the proposed iterative construction in Table I with large enough λ .

First, consider the case where $M_\alpha \geq M_\beta$. We build $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7d) as

$$[\mathbf{U}_{\beta 1}^T \cdots \mathbf{U}_{\beta L}^T]^T = (\mathbf{H}_\beta'^\dagger \mathbf{H}_\beta')^{-1} \mathbf{H}_\beta'^\dagger$$

where

$$\mathbf{H}_\beta' = [\mathbf{H}_{\beta 1} \mathbf{V}_{\beta 1} \quad \mathbf{H}_{\beta 2} \mathbf{V}_{\beta 2} \quad \cdots \quad \mathbf{H}_{\beta L} \mathbf{V}_{\beta L}].$$

After building $\{\mathbf{U}_{\beta l}\}_{l=1}^L$, we construct $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ satisfying (7b) and (7c) such that $[\mathbf{V}_{\alpha 1} \cdots \mathbf{V}_{\alpha K}]$ is set as the first $\sum_{k=1}^K d_{\alpha k}$ column vectors of $\mathbf{H}_\alpha'^\dagger (\mathbf{H}_\alpha' \mathbf{H}_\alpha')^{-1}$, where

$$\mathbf{H}_\alpha' = \begin{bmatrix} \mathbf{U}_{\alpha 1}^\dagger \mathbf{H}_{\alpha 1} \\ \vdots \\ \mathbf{U}_{\alpha K}^\dagger \mathbf{H}_{\alpha K} \\ [\mathbf{U}_{\beta 1}^T \cdots \mathbf{U}_{\beta L}^T]^T \mathbf{G}_\beta \end{bmatrix}$$

for given $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}, \mathbf{U}_{\beta l}\}_{l=1}^L$.

On the other hand, for $M_\alpha < M_\beta$, we first build $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ only satisfying (7c) as

$$[\mathbf{V}_{\alpha 1} \cdots \mathbf{V}_{\alpha K}] = \mathbf{H}_\alpha''^\dagger (\mathbf{H}_\alpha'' \mathbf{H}_\alpha'')^{-1}$$

where

$$\mathbf{H}_\alpha'' = [\mathbf{U}_{\alpha 1}^\dagger \mathbf{H}_{\alpha 1} \quad \mathbf{U}_{\alpha 2}^\dagger \mathbf{H}_{\alpha 2} \quad \cdots \quad \mathbf{U}_{\alpha K}^\dagger \mathbf{H}_{\alpha K}].$$

Then construct $\{\mathbf{U}_{\beta l}\}_{l=1}^L$ satisfying (7b) and (7d) such that $[\mathbf{U}_{\beta 1}^T \cdots \mathbf{U}_{\beta L}^T]^T$ is set as the first $\sum_{l=1}^L d_{\beta l}$ row vectors of $(\mathbf{H}_{\beta}^{\prime\prime\top} \mathbf{H}_{\beta}^{\prime\prime})^{-1} \mathbf{H}_{\beta}^{\prime\prime\top}$, where

$$\mathbf{H}_{\beta}^{\prime\prime} = \begin{bmatrix} \mathbf{H}_{\beta 1} \mathbf{V}_{\beta 1} & \cdots & \mathbf{H}_{\beta L} \mathbf{V}_{\beta L} & \mathbf{G}_{\beta} [\mathbf{V}_{\alpha 1} & \cdots & \mathbf{V}_{\alpha K}] \end{bmatrix}.$$

Finally, for both cases, we normalize the constructed $\{\mathbf{V}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{U}_{\beta l}\}_{l=1}^L$. Specifically, we properly scale each column vector of $\mathbf{U}_{\beta l}$ (and $\mathbf{V}_{\alpha k}$) such that the squared norm of that column vector is equal to one.

C. Achievable Sum Rate

In this subsection, we derive the achievable sum rate by applying the precoding and postcoding matrices stated in Sections V-A and V-B. Let the covariance matrix for the desired streams of user (α, k) as

$$\mathbf{C}_{\alpha k}^{\text{desire}} = \frac{P_{\alpha k}}{d_{\alpha k}} \mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha k} (\mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha k})^{\dagger}.$$

Also define

$$\begin{aligned} \mathbf{C}_{\alpha k}^{\text{inter}} &= \sum_{l=1}^L \frac{P_{\beta l}}{d_{\beta l}} \mathbf{U}_{\alpha k}^{\dagger} \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l} (\mathbf{U}_{\alpha k}^{\dagger} \mathbf{G}_{\alpha k l} \mathbf{V}_{\beta l})^{\dagger}, \\ \mathbf{C}_{\alpha k}^{\text{intra}} &= \sum_{i=1, i \neq k}^K \frac{P_{\alpha i}}{d_{\alpha i}} \mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha i} (\mathbf{U}_{\alpha k}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\alpha i})^{\dagger}, \end{aligned}$$

each of which are the inter-cell and intra-cell interference covariance matrices at user (α, k) . Then the achievable rate of user (α, k) is given by

$$R_{\alpha k} = \log_2 \det (\mathbf{I}_{d_{\alpha k}} + \mathbf{C}_{\alpha k}^{\text{desire}} (\mathbf{I}_{d_{\alpha k}} + \mathbf{C}_{\alpha k}^{\text{intra}} + \mathbf{C}_{\alpha k}^{\text{inter}})^{-1}). \quad (29)$$

Similarly, the achievable rate of user (β, l) is given by

$$R_{\beta l} = \log_2 \det (\mathbf{I}_{d_{\beta l}} + \mathbf{C}_{\beta l}^{\text{desire}} (\mathbf{I}_{d_{\beta l}} + \mathbf{C}_{\beta l}^{\text{intra}} + \mathbf{C}_{\beta l}^{\text{inter}})^{-1}) \quad (30)$$

where

$$\begin{aligned} \mathbf{C}_{\beta l}^{\text{desire}} &= \frac{P_{\beta l}}{d_{\beta l}} \mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\beta l} (\mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\alpha k} \mathbf{V}_{\beta l})^{\dagger}, \\ \mathbf{C}_{\beta l}^{\text{intra}} &= \sum_{i=1, i \neq l}^L \frac{P_{\beta i}}{d_{\beta i}} \mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\beta l} \mathbf{V}_{\beta i} (\mathbf{U}_{\beta l}^{\dagger} \mathbf{H}_{\beta l} \mathbf{V}_{\beta i})^{\dagger}, \\ \mathbf{C}_{\beta l}^{\text{inter}} &= \sum_{k=1}^K \frac{P_{\alpha k}}{d_{\alpha k}} \mathbf{U}_{\beta l}^{\dagger} \mathbf{G}_{\beta l} \mathbf{V}_{\alpha k} (\mathbf{U}_{\beta l}^{\dagger} \mathbf{G}_{\beta l} \mathbf{V}_{\alpha k})^{\dagger}. \end{aligned}$$

From (29) and (30), the achievable sum rate is given by

$$R_{\text{sum}} = \sum_{k=1}^K R_{\alpha k} + \sum_{l=1}^L R_{\beta l}. \quad (31)$$

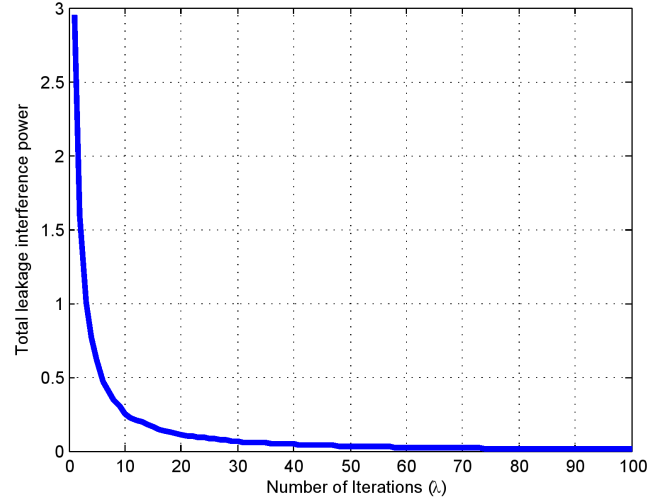


Fig. 4. Total leakage interference power of the $(12, (8, 8, 8, 8)) \times (18, (4, 4, 4))$ MIMO R-TDD cellular network.

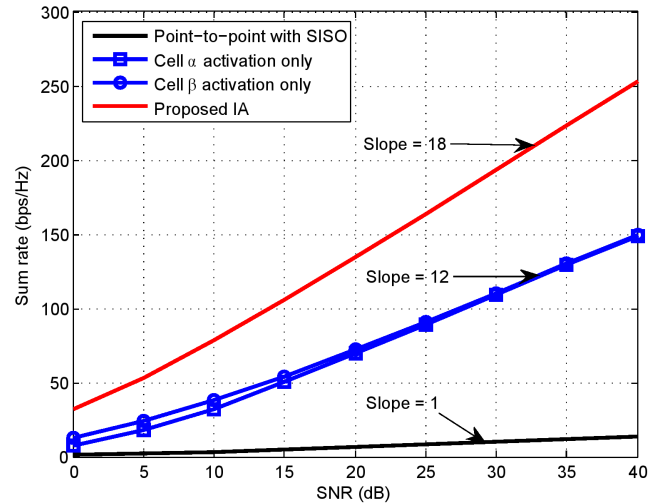


Fig. 5. Sum rate of the $(12, (8, 8, 8, 8)) \times (18, (4, 4, 4))$ MIMO R-TDD cellular network with feasible sum DoF of $d_{\text{sum}} = 18$.

D. Simulation

In order to demonstrate the sum rate improvement achievable by the proposed construction at the finite SNR regime, we simulate the sum rate in Section V-C for the $(12, (8, 8, 8, 8)) \times (18, (4, 4, 4))$ MIMO R-TDD cellular network. For this case, $d_{\alpha k} = 3$ for all $k \in [1 : 4]$ and $d_{\beta l} = 2$ for all $l \in [1 : 3]$ is feasible, which is used for simulation. In simulation, we assume that channel coefficients are i.i.d. drawn from $\mathcal{CN}(0, 1)$. We then average out the performance over large enough channel realizations.

Figure 4 plots the total leakage interference power $I^{[\lambda]}$ with respect to the number of iterations λ . As shown in the figure, $I^{[\lambda]}$ quickly converges to zero as λ increases, showing that the proposed iterative construction for $\{\mathbf{U}_{\alpha k}\}_{k=1}^K$ and $\{\mathbf{V}_{\beta l}\}_{l=1}^L$ works well for a moderate number of iterations.

Figure 5 plots R_{sum} in (31) with respect to SNR in dB scale. Specifically, we set $P_{\alpha} = \text{SNR}$ and $P_{\beta l} = \text{SNR}$ for

all $l \in [1 : L]$. For comparison, we also plot the sum rate achievable by activating either cell α or cell β only and the point-to-point capacity with single-antenna nodes. We assume zero-forcing precoding and postcoding matrices for single-cell activation. For this system, optimal sum DoF is achieved by this proposed IA since Theorem 1 implies $d_{\text{sum}} \leq 18$. The result demonstrates that the proposed IA not only provides the optimal sum DoF but also significantly enhance the sum rate at the finite SNR regime.

VI. CONCLUDING REMARK

In this paper, we established a necessary condition and a sufficient condition on IA feasibility for MIMO R-TDD cellular networks, which characterize the optimal sum DoF for many practical network configurations. Our results demonstrate that interference can be effectively mitigated by the proposed one-shot linear IA using multiple antennas. Although D-TDD or R-TDD systems have been actively studied, especially for heterogeneous cellular networks, there is still many remaining issues associated with IA in R-TDD systems. This work maybe extended to further arbitrary number of cells in R-TDD cellular networks. For a practical deployment, an efficient IA scheme for heterogeneous R-TDD cellular networks needs to be studied further.

REFERENCES

- [1] D. Astely, E. Dahlman, G. Fodor, S. Parkvall, and J. Sachs, "LTE release 12 and beyond," *IEEE Commun. Mag.*, vol. 51, no. 7, pp. 154–160, Jul. 2013.
- [2] "Cisco visual networking index: Global mobile data traffic forecast update, 2013-2018," White Paper, Cisco, Feb. 2014. [Online]. Available: <http://www.cisco.com/>
- [3] "3GPP TR 36.842, Study on small cell enhancements for E-UTRA and E-UTRAN," 3GPP, Dec. 2013. [Online]. Available: <http://www.3gpp.org/>
- [4] "3GPP TR 36.839, Mobility enhancements in heterogeneous networks," 3GPP, Sep 2012. [Online]. Available: <http://www.3gpp.org/>
- [5] M. Kountouris and N. Pappas, "HetNets and massive MIMO: Modeling, potential gains, and performance analysis," in *Proc. IEEE Antennas and Propagation in Wireless Communications (APWC)*, Torino, Italy, Sep. 2013.
- [6] J. Andrews, "Seven ways that HetNets are a cellular paradigm shift," *IEEE Commun. Mag.*, vol. 51, no. 3, pp. 136–144, Mar. 2013.
- [7] K. Hosseini, J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO and small cells: How to densify heterogeneous networks," in *Proc. IEEE International Conference on Communications (ICC)*, Budapest, Hungary, Jun. 2013.
- [8] A. Adhikary, E. Al Safadi, and G. Caire, "Massive MIMO and inter-tier interference coordination," in *Proc. Information Theory and Applications Workshop*, San Diego, CA, Feb. 2014.
- [9] A. Ghosh, N. Mangalvedhe, R. Ratasuk, B. Mondal, M. Cudak, E. Vitsosky, T. Thomas, J. Andrews, P. Xia, H. Jo, H. Dhillon, and T. Novlan, "Heterogeneous cellular networks: From theory to practice," *IEEE Commun. Mag.*, vol. 50, no. 6, pp. 54–64, Jun. 2012.
- [10] W. Jeong and M. Kavehrad, "Cochannel interference reduction in dynamic-TDD fixed wireless applications, using time slot allocation algorithms," *IEEE Trans. Commun.*, vol. 50, no. 10, pp. 1627–1636, Oct. 2002.
- [11] J. Yun and M. Kavehrad, "Adaptive resource allocations for D-TDD systems in wireless cellular networks," in *Proc. IEEE MILCOM*, Monterey, CA, Oct. 2004.
- [12] I. Sohn, K.-B. Lee, and Y. Choi, "Comparison of decentralized time slot allocation strategies for asymmetric traffic in TDD systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2990–3003, Jun. 2009.
- [13] H. Ji, Y. Kim, S. Choi, J. Cho, and J. Lee, "Dynamic resource adaptation in beyond LTE-A TDD heterogeneous networks," in *Proc. IEEE International Conference on Communications (ICC)*, Budapest, Hungary, Jun. 2013.
- [14] H. Sun, M. Wildemeersch, M. Sheng, and T. Q. S. Quek, "D2D enhanced heterogeneous cellular networks with dynamic TDD," in *arXiv:cs.NI/1406.2752*, Jun. 2014.
- [15] D. Chatterjee, J. Fwu, A. Khoryaev, H. He, M. Shilov, A. Chervyakov, and S. Pantelev, "Adaptive ul-dl tdd configurations in a heterogeneous network," U.S. Patent 20130272170, Oct. 17, 2013.
- [16] "3GPP TR 36.828, Further enhancements to LTE time division duplex (TDD) for downlink-uplink (DL-UL) interference management and traffic adaptation," 3GPP, Jun. 2012. [Online]. Available: <http://www.3gpp.org/>
- [17] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K -user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
- [18] —, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3893–3908, Sep. 2009.
- [19] C. Suh, M. Ho, and D. Tse, "Downlink interference alignment," *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2616–2626, Sep. 2011.
- [20] C. Suh and D. Tse, "Interference alignment for cellular networks," in *Proc. 46th Annu. Allerton Conf. Communication, Control, and Computing*, Monticello, IL, Sep. 2008.
- [21] V. Cadambe and S. Jafar, "Degrees of freedom of wireless networks with relays, feedback, cooperation, and full duplex operation," *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2334–2344, May 2009.
- [22] S.-W. Jeon and M. Gastpar, "A survey on interference networks: Interference alignment and neutralization," *Entropy*, vol. 14, no. 10, pp. 1842–1863, Sep. 2012.
- [23] L. Ke, A. Ramamoorthy, Z. Wang, and H. Yin, "Degrees of freedom region for an interference network with general message demands," *IEEE Trans. Inf. Theory*, vol. 58, no. 6, pp. 3787–3797, Jun. 2012.
- [24] T. Gou, S. Jafar, C. Wang, S.-W. Jeon, and S.-Y. Chung, "Aligned interference neutralization and the degrees of freedom of the $2 \times 2 \times 2$ interference channel," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4381–4395, Jul. 2012.
- [25] B. Nazer, M. Gastpar, S. Jafar, and S. Vishwanath, "Ergodic interference alignment," *IEEE Trans. Inf. Theory*, vol. 58, no. 10, pp. 6355–6371, Oct. 2012.
- [26] S.-W. Jeon and S.-Y. Chung, "Capacity of a class of linear binary field multisource relay networks," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6405–6420, Oct. 2013.
- [27] S.-W. Jeon, C.-Y. Wang, and M. Gastpar, "Approximate ergodic capacity of a class of fading two-user two-hop networks," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 866–880, Feb. 2014.
- [28] S.-W. Jeon, S.-Y. Chung, and S. Jafar, "Degrees of freedom region of a class of multisource Gaussian relay networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3032–3044, May 2011.
- [29] A. S. Motahari, S. O. Gharan, and A. K. Khandani, "Real interference alignment with real numbers," in *arXiv:cs.IT/0908.1208*, 2009.
- [30] A. Motahari and A. Khandani, "Capacity bounds for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 620–643, Feb. 2009.
- [31] S. Jafar and M. Fakhereddin, "Degrees of freedom for the MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 53, no. 7, pp. 2637–2642, Jul. 2007.
- [32] T. Gou and S. Jafar, "Degrees of freedom of the K user $M \times N$ MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.
- [33] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "Feasibility conditions for interference alignment," in *Proc. IEEE GLOBECOM*, Honolulu, HI, Nov. 2009.
- [34] —, "On feasibility of interference alignment in MIMO interference networks," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.
- [35] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, "On the degrees of freedom achievable through interference alignment in a MIMO interference channel," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 812–821, Feb. 2012.
- [36] L. Ruan, V. Lau, and M. Win, "The feasibility conditions for interference alignment in MIMO networks," *IEEE Trans. Signal Process.*, vol. 61, no. 8, pp. 2066–2077, Apr. 2013.
- [37] G. Bresler, D. Cartwright, and D. Tse, "Feasibility of interference alignment for the mimo interference channel," *IEEE Trans. Inf. Theory*, vol. 60, no. 9, pp. 5573–5586, Sep. 2014.
- [38] T. Kim, D. J. Love, and B. Clerckx, "On the spatial degrees of freedom of multicell and multiuser MIMO channels," in *arXiv:1111.3160*, Nov. 2011.

- [39] D. Hwang, "Interference alignment for the multi-cell multiuser interference channel," *IEEE Commun. Lett.*, vol. 16, no. 6, pp. 831–833, Jun. 2012.
- [40] S. Ayoughi, M. Nasiri-Kenari, and B. Khalaj, "On degrees of freedom of the cognitive MIMO two-interfering multiple-access channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 5, pp. 2052–2068, Jun. 2013.
- [41] V. Ntranos, M. A. Maddah-Ali, and G. Caire, "Cellular interference alignment," in *arXiv:cs.IT/1404.6512*, Apr. 2014.
- [42] M. Guillaud and D. Gesbert, "Interference alignment in partially connected interfering multiple-access and broadcast channels," in *Proc. IEEE GLOBECOM*, New Orleans, LA, Dec. 2011.
- [43] Y. Ma, J. Li, R. Chen, and Q. Liu, "On feasibility of interference alignment for l -cell constant cellular interfering networks," *IEEE Commun. Lett.*, vol. 16, no. 5, pp. 714–716, May 2012.
- [44] T. Liu and C. Yang, "On the feasibility of linear interference alignment for MIMO interference broadcast channels with constant coefficients," *IEEE Trans. Signal Process.*, vol. 61, no. 9, pp. 2178–2191, May 2013.
- [45] —, "On the degrees of freedom of asymmetric MIMO interference broadcast channels," in *arXiv:cs.IT/1310.7311*, Oct. 2013.
- [46] B. Zhuang, R. Berry, and M. Honig, "Interference alignment in MIMO cellular networks," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Prague, Czech Republic, May 2011.
- [47] J. Tang and S. Lambotharan, "Interference alignment techniques for mimo multi-cell interfering broadcast channels," *IEEE Trans. Commun.*, vol. 61, no. 1, pp. 164–175, Jan. 2013.
- [48] W. Shin, N. Lee, J.-B. Lim, C. Shin, and K. Jang, "On the design of interference alignment scheme for two-cell MIMO interfering broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 437–442, Feb. 2011.
- [49] J. Shin and J. Moon, "Regularized zero-forcing interference alignment for the two-cell MIMO interfering broadcast channel," *IEEE Commun. Lett.*, vol. 17, no. 7, pp. 1336–1339, Jul. 2013.
- [50] G. Sridharan and W. Yu, "Degrees of freedom of MIMO cellular networks: Decomposition and linear beamforming design," in *arXiv:cs.IT/1312.2681*, Dec. 2013.
- [51] W. Shin, W. Noh, K. Jang, and H.-H. Choi, "Hierarchical interference alignment for downlink heterogeneous networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4549–4559, Dec. 2012.
- [52] S.-W. Jeon and C. Suh, "Degrees of freedom of uplink–downlink multiantenna cellular networks," in *arXiv:cs.IT/1404.6012*, Apr. 2014.
- [53] K. Gomadam, V. Cadambe, and S. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.
- [54] D. West, *Introduction to Graph Theory*. Prentice Hall, 2001.